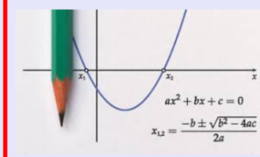
Math 125

Spring 2022

Lecture 15



Class QZ 12

Solve for $\boxed{x \text{ only}}$ using Cramer's rule.

$$\begin{cases} 4x - 3y = 5 \\ x + y = 3 \end{cases}$$

$$D = \begin{vmatrix} 4 & -3 \\ 1 & 1 \end{vmatrix} = 4(1) - 1(-3) = \boxed{7}$$

$$D_x = \begin{vmatrix} 5 & -3 \\ 3 & 1 \end{vmatrix} = 5(1) - 3(-3) = \boxed{14}$$

$$x = \frac{D_x}{D} = \frac{14}{7} = \boxed{2} \qquad \boxed{x=2}$$

$$\begin{cases} 4x - 3y = 5 \\ x + y = 3 \end{cases}$$

Solve by matrix Method:

$$\left[\begin{array}{cc|c} 4 & -3 & 5 \\ 1 & 1 & 3 \end{array}\right]$$

Augmented Matrix

$R_1 \leftrightarrow R_2$ $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 4 & -3 & 5 \end{array}\right]$

$(-4)R_1 + R_2 \to R_2$ $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -7 & -7 \end{array}\right]$

$R_2 \div (-7) \to R_2$ $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 1 \end{array}\right]$

$(-1)R_2 + R_1 \to R_1$ $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right] \begin{array}{l} \Rightarrow x = 2 \\ \Rightarrow y = 1 \end{array}$

Final Ans: $(2,1)$

Soln Set: $\{(2,1)\}$

Use Cramer's rule to solve for $\boxed{z \text{ only}}$:

$$\begin{cases} x + y - z = -2 \\ 2x - y + z = 5 \\ -x + 2y + 2z = 1 \end{cases}$$

$$z = \frac{D_z}{D} = \frac{-24}{-12} = 2$$

$$\boxed{z = 2}$$

Always

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ -1 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= 1(-2-2) - 1(4+1) - 1(4-1)$$

$$= -4 - 5 - 3 = \boxed{-12}$$

$$D_z = \begin{vmatrix} 1 & 1 & -2 \\ 2 & -1 & 5 \\ -1 & 2 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 & 5 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & 5 \\ -1 & 1 \end{vmatrix} + (-2)\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= 1(-1-10) - 1(2+5) - 2(4-1)$$

$$= -11 - 7 - 6 = \boxed{-24}$$

March 29, 2022

Solve by matrix Method:

$$\begin{cases} x + y - z = -2 \\ 2x - y + z = 5 \\ -x + 2y + 2z = 1 \end{cases}$$

Augmented Matrix

Pivot

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right]$$

$(-2)R_1 + R_2 \to R_2$

$R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{array}\right]$$

$R_2 \div (-3) \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 3 & 1 & -1 \end{array}\right]$$

$(-3)R_2 + R_3 \to R_3$

$(-1)R_2 + R_1 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

$R_3 \div 4 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_3 + R_2 \to R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} \Rightarrow x = 1 \\ \Rightarrow y = -1 \\ \Rightarrow z = 2 \end{array}$$

Final Ans: $(1, -1, 2)$

Soln Set: $\{(1, -1, 2)\}$

Support Services:
1) SI
2) Tutoring Lab
3) DSPS
4) Lecture notes & Lecture Videos
5) office hours
6) Peers

Solve by Matrix Method:

$$\begin{cases} 3x - 6y = 1 \\ 2x - 4y = \frac{2}{3} \end{cases}$$

1) Augmented Matrix

$$\left[\begin{array}{cc|c} 3 & -6 & 1 \\ 2 & -4 & \frac{2}{3} \end{array}\right]$$

$2R_1 \to R_1$
$3R_2 \to R_2$

$$\left[\begin{array}{cc|c} 6 & -12 & 2 \\ 6 & -12 & 2 \end{array}\right]$$

$(-1)R_1 + R_2 \to R_2$

$$\left[\begin{array}{cc|c} 6 & -12 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

when an entire row becomes zeros

There are infinite # of Solutions

$$\{(x,y) \mid 3x - 6y = 1\}$$

Solve by matrix Method:

$$\begin{cases} 4x - 2y = 5 \\ -2x + y = 6 \end{cases}$$

$$\left[\begin{array}{cc|c} 4 & -2 & 5 \\ -2 & 1 & 6 \end{array}\right]$$

$(2)R_2 \to R_2$

$$\left[\begin{array}{cc|c} 4 & -2 & 5 \\ -4 & 2 & 12 \end{array}\right]$$

$R_1 + R_2 \to R_2$

$$\left[\begin{array}{cc|c} 4 & -2 & 5 \\ 0 & 0 & 17 \end{array}\right]$$

when an entire row becomes zeros except the last number $\Rightarrow$ There is No Solution.

$$\emptyset$$

Class QZ 13

Evaluate by expanding about first row:

$$\begin{vmatrix} \boxed{1 & -2 & 5} \\ 3 & 1 & 2 \\ 4 & -1 & 7 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ -1 & 7 \end{vmatrix} - (-2)\begin{vmatrix} 3 & 2 \\ 4 & 7 \end{vmatrix} + 5\begin{vmatrix} 3 & 1 \\ 4 & -1 \end{vmatrix}$$

$$= 1(7+2) + 2(21-8) + 5(-3-4)$$

$$= 9 + 2(13) + 5(-7)$$

$$= 9 + 26 - 35 = 35 - 35$$

$$= \boxed{0}$$

Solve by subs. method:

Non linear system of equations

$$\begin{cases} x^2 = 2y + 10 \\ y = 3x - 9 \end{cases}$$

$x^2 = 2(3x - 9) + 10$

$x^2 = 6x - 18 + 10$

$x^2 = 6x - 8$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

Zero-Product Rule

$x - 4 = 0$ $\quad$ $x - 2 = 0$

$x = 4$ $\quad$ $x = 2$

when $x = 4$

$y = 3x - 9 = 3(4) - 9$

$= 12 - 9$

$= 3$

$(4, 3)$

when $x = 2$

$y = 3x - 9 = 3(2) - 9$

$= 6 - 9$

$= -3$

$(2, -3)$

Soln Set $\{(4,3), (2,-3)\}$

Solve by Subs. method:

$$\begin{cases} x^2 = y - 1 \\ 4x - \boxed{y} = -1 \end{cases} \Rightarrow \boxed{x^2+1} = y$$

$$4x - (x^2+1) = -1 \qquad 4x - x^2 \cancel{-1} = \cancel{-1}$$

$$4x - x^2 = 0$$

$$x(4-x) = 0$$

$$x = 0 \qquad 4 - x = 0$$

$$\boxed{x=0} \qquad \boxed{x=4}$$

when $x=0$

$y = 0^2 + 1 \qquad y = 1$

$(0,1)$

If $x=4$

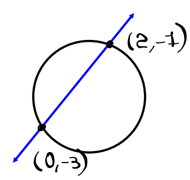
$y = 4^2 + 1 \qquad y = 17$

$(4,17)$

$$\{(0,1),(4,17)\}$$

Solve by Subs. Method:

$$\begin{cases} x - y = 3 \\ (x-2)^2 + (y+3)^2 = 4 \end{cases} \Rightarrow \boxed{x = \boxed{y+3}}$$



$$(y+3-2)^2 + (y+3)^2 = 4$$

$$(y+1)^2 + (y+3)^2 = 4$$

$$(y+1)(y+1) + (y+3)(y+3) = 4$$

$$y^2 + y + y + 1 + y^2 + 3y + 3y + 9 - 4 = 0$$

$$2y^2 + 8y + 6 = 0$$

Divide by 2

$$y^2 + 4y + 3 = 0$$

$$(y+1)(y+3) = 0$$

$$y+1=0 \qquad y+3=0$$

$$y=-1 \qquad y=-3$$

If $y=-1$

$x = y+3 = -1+3 = 2$

$(2,-1)$

If $y=-3$

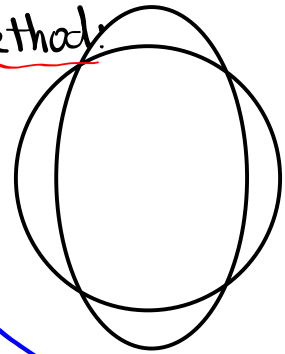
$x = y+3 = -3+3 = 0$

$(0,-3)$

Final Ans: $\{(2,-1),(0,-3)\}$

Solve by addition method:

$$\begin{cases} 4x^2 + y^2 = 13 \\ x^2 + y^2 = 10 \end{cases} \quad -1$$



$$\begin{cases} 4x^2 + \cancel{y^2} = 13 \\ -x^2 - \cancel{y^2} = -10 \end{cases}$$

$$3x^2 = 3$$

$$x^2 = \frac{3}{3} \qquad x^2 = 1 \Rightarrow \boxed{x = \pm 1}$$

$$x^2 + y^2 = 10$$
$$1 + y^2 = 10$$
$$y^2 = 10 - 1$$
$$y^2 = 9$$
$$\boxed{y = \pm 3}$$

$$\{(1,3), (1,-3), (-1,3), (-1,-3)\}$$

4 Answers

Solve by addition Method:

$$\begin{array}{r} 3 \\ -2 \end{array} \begin{cases} 3x^2 + 2y^2 = 35 \\ 4x^2 + 3y^2 = 48 \end{cases} \Rightarrow \begin{cases} 9x^2 + \cancel{6y^2} = 105 \\ -8x^2 - \cancel{6y^2} = -96 \end{cases}$$

$$x^2 = 9$$

$$x^2 = 9 \Rightarrow \boxed{x = \pm 3}$$

$$4(9) + 3y^2 = 48$$
$$36 + 3y^2 = 48$$
$$3y^2 = 48 - 36$$
$$3y^2 = 12 \rightarrow y^2 = 4 \Rightarrow \boxed{y = \pm 2}$$

$$\{(3,2), (3,-2), (-3,2), (-3,-2)\}$$

Solve:

$$\begin{cases} y = x^2 + 3 \\ x^2 + y^2 = 9 \end{cases} \Rightarrow \begin{cases} \cancel{-x^2} + y = 3 \\ \cancel{x^2} + y^2 = 9 \end{cases}$$

$$y^2 + y = 12$$

$$y^2 + y - 12 = 0$$

$$(y+4)(y-3) = 0$$

$y+4=0$ $\quad$ $y-3=0$

$y=-4$ $\quad$ $y=3$

$y = x^2 + 3$

If $y = 3$

$3 = x^2 + 3 \Rightarrow x^2 = 0$

$\Rightarrow \boxed{x=0}$

$(0,3)$

If $y=-4$

$-4 = x^2 + 3 \Rightarrow x^2 = -7$ $\quad$ No real Solutions

$\{(0,3)\}$

The Sum of two numbers is 10.

$\hookrightarrow x \ \& \ y$

Their product is 24.

Find both numbers.

$$\begin{cases} x + y = 10 \\ x \cdot y = 24 \end{cases}$$

$y = 10 - x$

$x(10-x) = 24$

$10x - x^2 = 24$

$-x^2 + 10x - 24 = 0$

Multiply by -1

$x^2 - 10x + 24 = 0$

$(x-4)(x-6) = 0$

$x-4=0$ $\quad$ $x-6=0$

$x=4$ $\quad$ $x=6$

$y = 10 - x$

If $x=4 \Rightarrow y = 10-4 = 6 \Rightarrow 4 \ \& \ 6$

If $x=6 \Rightarrow y = 10-6 = 4 \Rightarrow 6 \ \& \ 4$

The numbers are 4 & 6

March 29, 2022

The difference between squares of two numbers is 3. $x^2 - y^2 = 3$

Twice the square of first number increased by square of the second number is 9. $2x^2 + y^2 = 9$

Find the numbers.

$$\begin{cases} x^2 - \cancel{y^2} = 3 \\ 2x^2 + \cancel{y^2} = 9 \end{cases}$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$\boxed{x = \pm 2}$$

$$x^2 - y^2 = 3$$

$$4 - y^2 = 3$$

$$-y^2 = 3 - 4$$

$$-y^2 = -1 \quad y^2 = 1$$

$$\boxed{y = \pm 1}$$

$$\{(2,1),(2,-1),(-2,1),(-2,-1)\}$$

2 & 1, 2 & -1, -2 & 1, -2 & -1

Class QZ 14

Solve by **matrix Method**:

$$\begin{cases} x - 2y = 5 \\ 2x + 3y = -4 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 2 & 3 & -4 \end{array}\right]$$

$(-2)R_1 + R_2 \to R_2$

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 7 & -14 \end{array}\right]$$

$R_2 \div 7 \to R_2$

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -2 \end{array}\right] \begin{array}{l} \Rightarrow x = 1 \\ \Rightarrow y = -2 \end{array} \Rightarrow \boxed{(1,-2)}$$

$(2)R_2 + R_1 \to R_1$

$$\{(1,-2)\}$$